Cognitive biases arise in conflating epistemic and aleatory uncertainty

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Probability is hard

• Probability is a very young discipline
  – Although math is 22 centuries old, probability is only 2 or 3
  – Only invented for resolving games of chance

• Probability is famously counterintuitive
  – Monty Hall problem embarrassed prominent scholars
  – Experts (even Laplace) make egregious mistakes
  – De Morgan left probability because it was too hard
  – Rife with paradoxes, unlike any other branch of math
Probability paradoxes

- Ellsberg paradox
- St. Petersburg paradox
- Two-envelopes problem
- Monty Hall problem
- Simpson’s paradox
- Bertand paradox
- Berkson’s paradox
- Sleeping Beauty problem

“probability that a random chord of a circle is longer than a side of a triangle inscribed within the circle” is not well defined

two independent events become conditionally dependent
(negatively dependent) given that at least one of them occurs

A sleeper is wakened once or twice according to the toss of a coin, but wouldn’t remember a previous waking, if any. What should be her credence that the coin came up heads?
Cognitive biases

• Psychometry describes cognitive biases that make humans prone to errors because of the way our brains are wired
  – Groupthink
  – Stereotyping
  – Memory flaws
  – Illusions of control

• Kahneman and Tversky reviewed many such biases in how humans perceive risks and uncertainties and make decisions
Decision biases

- **Loss aversion**
  Disliking a loss more fervently than liking a gain of the same magnitude

- **Ambiguity aversion**
  Avoiding options when probabilities seem unknown

- **Zero-risk bias**
  Preferring to reduce a small risk to zero over a greater reduction in a larger risk

- **Anchoring**
  Relying too heavily on a past reference or one piece of information

- **Availability heuristic**
  Estimating likelihood of something by the ease with which it’s remembered
Uncertainty biases

• Probability misperception
  Overestimating chance of rare outcomes, understating chances of common ones

• Conjunction fallacy
  Assuming that specific conditions are more probable than general ones

• Pseudocertainty
  Making risk-averse choices for positive outcomes, but risk-seeking for negative

• Overconfidence
  Excessive confidence in one’s own predictions

• Base rate fallacy
  Neglecting available statistical data in favor of particulars

• Neglect of probability
  Disregarding probability in decision making under uncertainty
Other biases

- Clustering illusion
  Seeing patterns in noise
- Ludic fallacy
  Believing that chance in life is like chance in games
- Primacy
  Weighting initial events more than subsequent events
- Recency
  Weighting recent events more than earlier events
- Gambler’s fallacy
  Thinking future probabilities are altered by past events, e.g., P(head | 4 tails)
- Framing
  Drawing different conclusions based on how data are presented
- Regression toward the mean
  Expecting extreme performance to continue
- Hyperbolic discounting
  Strongly preferring immediate payoffs over later
Heuristics

• Biases are presumed to be the result of using imperfect mental shortcuts, called “heuristics”

• Humans’ misconceptions are the results of bad wiring in our brains

• And people are especially stupid about risks and uncertainty
How can this be?

• But then, how have humans been so successful evolutionarily?

• Risks and uncertainty was surely important in human history

• Luce: the main finding of decision theory is that humans don’t make decisions like decision theory says they should
Why is risk communication hard?

Experts often say the public is irrational

But maybe what experts have been telling people is incomprehensible or irrelevant

What people are evolved to comprehend?
Neuroscience of risk perception

Decade of the Brain (’90s)
Neuroscience of risk perception

Instead of being divided into rational and emotional sides, the human brain has *many* special-purpose calculators

(Marr 1982; Barkow et al. 1992; Pinker 1997, 2002)

Image by Dwayne Reed from an MRI at Stony Brook University Medical Center
Mental calculators  
(after Pinker 2002)

- Language (grammar and memorized dictionary)
- Practical physics (pre-Newtonian)
- Intuitive biology (animate differs from inanimate)
- Intuitive engineering (tools designed for a purpose)
- Spatial sense (dead reckoner and mental maps)
- Number sense (1, 2, 3, many)
- Probability sense (frequentist Bayes)
- Uncertainty detection (procrastination)
- Intuitive economics (reciprocity, trust, equity, fairness)
- Intuitive psychology (theory of mind, deception)
Some of the mental calculators
(after Pinker 2002)

- Number sense (1, 2, 3, many)
- Probability sense (frequentist Bayes)
- Uncertainty detection (procrastination)
- Intuitive economics (reciprocity, trust, fairness)
Probability sense

• We can watch the probability calculator turn on

• Platt and Glimcher found neurons in the lateral intraparietal cortex in rhesus monkeys encode an outcome’s probability and magnitude

• We can also see it in reasoning behaviors

Bayesian reasoning (poor)

If a test to detect a disease whose prevalence is 0.1% has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person’s symptoms or signs? ___%

12-18% correct

Casscells et al. 1978 replicated in Cosmides and Tooby 1996
Bayesian reasoning (good)

If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 50/1000, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person’s symptoms or signs? __1__ out of __51__.

8 or 9 out 10 correct

Casscells et al. 1978 replicated in Cosmides and Tooby 1996
A calculator must be triggered

• Humans have an innate probability sense

• But it is triggered by *natural frequencies*

• The calculator kicked in for the students who got the question in terms of natural frequencies, and they mostly solved it

• The mere presence of the percent signs hobbled the other group
Multiple calculators may fire

- There are distinct calculators associated with
  - Probabilities and risk (variability)
  - Ambiguity and uncertainty (incertitude)
  - Trust and fairness

- Brain processes them differently
  - Different parts of the brain
  - Different chemical systems

- They can give conflicting responses
Risk aversion

• Suppose you can get $1000 if a randomly drawn ball is red from urn with half red and half blue balls, or you can just get $500 now

• Which prize do you want?

$500

EU is the same, but most people take the sure $500
Ambiguity aversion

- Balls can be either red or blue
- Two urns, both with 36 balls
- Get $1000 if a randomly drawn ball is red
- Which urn do you wanna draw from?

A probabilist could explain your preference by saying your probability for red in the opaque urn is low.
Ellsberg Paradox

• Balls can be red, black or yellow \( (\text{probs are } R, B, Y) \)
• A well-mixed urn has 30 red balls and 60 other balls
• Don’t know how many are black, how many are yellow

<table>
<thead>
<tr>
<th>Gamble A</th>
<th>( R &gt; B )</th>
<th>Gamble B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get $100 if draw red</td>
<td>Get $100 if draw black</td>
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<table>
<thead>
<tr>
<th>Gamble C</th>
<th>Gamble D</th>
<th>( R &lt; B )</th>
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<tbody>
<tr>
<td>Get $100 if red or yellow</td>
<td>Get $100 if black or yellow</td>
<td></td>
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Persistent paradox

• People always prefer unambiguous outcomes
  – Doesn’t depend on your utility function or payoff
  – Not related to risk aversion
  – We simply don’t like ambiguity

• Not explained by probability theory, or by prospect theory
Other species

- Chimpanzees preferred peanuts (which they like less than bananas) when they don’t know the probability of getting bananas.

Hsu et al. (2005) found localized brain activity regions under situations of ambiguity.

Amygdala processing fear and threat.

Ambiguity/incertitude detector

• Humans have an incertitude processor
  – Triggered by situations with ambiguity
  – Especially focused on the worst case
  – Common response is procrastination

• Functional organ
  – Normal feature of the human brain
  – Not a product of learning
  – Visible in fMRI

• Brain lesions can make people insensitive to incertitude…so they behave as rational Bayesians
Biological basis for Ellsberg

- Probability sense and the ambiguity detector interfere with each other

- Humans do not make decisions based purely on probability in such cases

- Probabilists use equiprobability to model incertitude which confounds it with variability

Hsu et al. 2005
Probability distortion
Humans distort probabilities

- People behave as though small risks are larger than they actually are, and large risks are smaller than they actually are.
- The over/under transition point is variable.
Different people differ broadly
Probability distortion function

• Key element of Prospect Theory \(^{Kaheman \ & \ Tversky \ 1989}\)

• No good explanation for it

• No explanation for inter-individual differences
Johnson and Luhmann

• This distortion is suboptimal and irrational
  – At least assuming empirical probabilities are accurate

• What if probabilities are imprecise (ambiguous)
  – What would be the optimal strategy?

• They used an agent-based simulation to find out
Agent-based simulation

- Agents given different distortion functions

- Agents make risky choices in a probabilistic environment with imperfect information

- Genetic algorithm identifies the optimal distortion function
Risky choices under evolution

- Binary choices 200 over each lifetime
- Randomly selected probabilities uniform(0,1)
- Randomly selected rewards uniform(10,1000)
- Agents received expected value of chosen gamble
- Imperfect information about probabilities sample size
- Top half of agents each generation get to reproduce
- Offspring get parent’s distortion parameter + noise
- More choices hastens convergence
Risky choice

Which gamble do you prefer?

- 64% chance of $75, EU = $48
- 8% chance of $409, EU = $33

You can do about 50% better by choosing the left.

If you get to make many such choices, go with EU.
Ambiguity about probability

• What the agents see is *not* the true probability
• Binomial distributions sample true probability
• Samples define probability given to agents
• Fewer samples $\rightarrow$ more ambiguity
• Seven sample sizes: 1000, 100, 50, 20, 10, 5, 3
True probabilities and payoffs

<table>
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<th>0.5</th>
<th>0.75</th>
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<tr>
<td>$100</td>
<td>$80</td>
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$60

Sampling from binomial distribution \((n = 10)\)

| 4/10 | 9/10 |

Choices given to agent (nominal probabilities)

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<th>0.4</th>
<th>0.9</th>
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<tbody>
<tr>
<td>$100</td>
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$72

Agent weights probabilities

<table>
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<th>0.43</th>
<th>0.79</th>
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<td>$100</td>
<td>$80</td>
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$63
Results

Evolution Example (Sample Size = 3)
Optimal weighting
Kelli and Christian’s conclusions

• Distortion is advantageous under ambiguity

• Best to overweight small probabilities and underweight large probabilities

• Same results with non-linear agents

• People *should* be sensitive to ambiguity

• Distortion may not be cognitive limitation
Loss aversion

(asymmetry in perceptions about losses and gains)
Prospect theory

People hate losses more than they love gains

Outcome

Value

Losses

Gains

Loss aversion
But why?

• Prospect theory is the state of the art
• Purely descriptive
• Doesn’t say *why* loss aversion should exist

• What is the biological basis for loss aversion?
• How could it have arisen in human evolution?
Let's make a simpler symmetry assumption.
If uncertainty is massive…

…the bottom falls out of the market

Gains seem valueless; Losses seem infinite
Loss aversion disappears with certainty

- Loss aversion disappears
  - with a person you trust, or
  - after the gamble has been realized
    - Gilbert et al. 2004
    - Kermer et al. 2006
    - Yechiam & Ert 2007
    - Erev, Ert, & Yechiam 2008
    - Ert & Erev 2008

- When losses and gains are surely exchangeable, the uncertainty contracts to the symmetric utility
Direct experimental evidence

• Ellsberg made the probabilities ambiguous

• Psychologist Christian Luhmann (Stony Brook) made rewards ambiguous
  – Visually obscured the promised payoffs
  – “I’ll pay you between 1 and 10 bucks”

• Loss aversion varies with the size of uncertainty
• Disappears with certainty
Clinical evidence

- Amygdala damage eliminates loss aversion

- But doesn’t affect a person’s ability to gamble and respond to changing value or risk ($n = 2$)

- Amygdalectomied rhesus monkeys approach stimuli that healthy monkeys avoid


But why pessimism?

- Pessimism is often advantageous evolutionarily
- Natural selection can favor pessimism
  - Death is ‘hard selection’
  - Animal foraging strategies
  - Programmed plant behaviors
- Being wrong often has asymmetric consequences
  - Foraging: Finding dinner versus being dinner
  - Competition: Preemption versus being preempted
And even in plants!

Same total amount of soil

Plant pessimism

• When grown together, plants make more roots
• Less efficient than what they do when alone

• Competition is asymmetric, first come first serve
• They both grow more roots than they need just to prevent being competitively preempted

• Tragedy of the commons / prisoners’ dilemma
Pessimism is not inevitable

• Pessimism is not the only reaction to uncertainty
  – Maniacs
  – Pathological gamblers
  – Normal people in stressful situations

• Ambiguity aversion decreases with optimism
  (Pulford 2009)

Collisions of the two cameras

• Ambiguity aversion (Ellsberg paradox)
• Probability neglect
• Loss aversion
• Framing effects
• Hyperbolic discounting
• Two-envelopes problem
• Slovic’s two-dimensional plot of risks

The third fairness calculator explains even more
Irrationality is a hallmark of human decisions

- Eat and drink things that make us sick
- Crave immediacy, even when delay is better
- Love people we shouldn’t
- Bury the dead, sometimes at great cost
- Retrieve fallen comrades from the battlefield
- Engage in spite and noblesse oblige
So why are humans “irrational”?

• Using the wrong mental calculator
  – Like an optical illusion

• Disagreement among mental calculators
  – Collisions one calculator wins, vacillation, confusion

• Concerned with issues outside the risk analysis
  • Justice
  • Fairness
  • Chance the risk analyst is lying
  • Chance the risk analyst is inept

Different calculators
Import for risk assessment

• Risk analyses woefully incomplete
  – Neglect or misunderstand incertitude
  – Omit important issues and thus understate risks

• Presentations use very misleading formatting
  – Percentages, relative frequencies, averages, conditionals, etc.

• Both problems can be fixed
  – By changing analysts’ behavior (not the public’s)